MO Frequency Stability Requirements for Coherent Ladar

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Introduction

Coherent ladar techniques are being considered for various commercial and military applications. These include high resolution velocity measurements, remote vibration measurements, optical synthetic aperture implementations, and other applications for which knowledge of signal phase and amplitude information is required.

In order to obtain high fidelity signal information, particularly for vibration or optical synthetic aperture measurements, the quality or fidelity of the transmitted signal must be very high. Because laser transmitters have finite coherence and experience frequency drift over time, they impose a limitation on the signal fidelity that can be achieved. Large time-bandwidth modulated transmitted waveforms also require a high degree of phase control and thus system components, such as modulators, can also limit the quality of return signal information.

Coherent ladar systems use heterodyne detection to sense the phase and frequency of the return signal and also to amplify it to improve the signal-to-noise ratio (SNR) of the return. With heterodyne detection the detector current is proportional to the product of the return and the local oscillator (LO) electric fields. As such, the signal will experience fluctuations from both the return and LO. Thus the frequency quality of the LO is as important as that of the transmitted waveform.

Heterodyne Detection Mathematics Using a Single Master Oscillator

In order to reduce the frequency stability requirements of a ladar, one usually derives the transmitter and local oscillator lasers from a common master oscillator (MO). There are a number of benefits from operating in this mode, however, it requires that any optical modulation used to generate the desired waveform is imparted by a modulator that is external to the laser resonator. This requirement limits the laser sources to wavelength regions where appropriate modulators are available. When both the transmitter and the LO laser are derived from a single master oscillator laser, the phase error term of the signal is

$$\phi_e(t) = \phi(t+\tau_0) - \phi(t), \qquad (1)$$

where τ_0 is the time difference between the return and LO beams at the detector. Often the time difference is given by the round-trip time to the target, or 2R/c where R is the target range and c is the speed of light. We are thus most concerned with phase fluctuations of the MO that occur within the time difference τ_0, and longer-term phase fluctuations are not as important. In other cases such as synthetic aperture imaging, the illuminating waveform can have long duration and phase fluctuations over long intervals are important.

To evaluate the impact of frequency errors, let us first consider the power spectrum, $S(\omega)$, which is given by[1],

$$S_\phi(\omega) = \left| \frac{1}{T} \int_{-T/2}^{+T/2} \phi(t) e^{-j\omega t} \cdot dt \right|^2 \quad \text{for T large.} \qquad (2)$$

The power spectrum for the phase error is

$$S_{\phi e}(\omega) = \left| \frac{1}{T} \int_{-T/2}^{+T/2} [\phi(t+\tau_0) - \phi(t)] e^{-j\omega t} \cdot dt \right|^2 . \qquad (3)$$

These expressions then allow us to write:

$$S_{\phi e}(\omega) = S_\phi(\omega) \cdot 4\left(\sin(\omega\tau_0/2)\right)^2 . \qquad (4)$$

Phase Noise Limitations and Frequency Stability for Synthetic Aperture Ladar (SAL)

For some coherent ladar applications such as SAL, the allowable phase error in the recorded signal is specified as the maximum allowable phase error. For other ladar systems such as vibration ladar, the maximum root-mean-squared (rms) frequency error is specified. In either case, the requirements on signal errors are used to specify the laser stability in terms of the rms frequency noise, which is a convenient laser parameter that can be measured and is often specified by laser manufacturers.

In Synthetic Aperture type imaging, the maximum phase error that can typically be tolerated is a fraction of 2π radians; $\pi/30$ is a typical value. Because the rms value of a random phase obeys random-walk statistics the error increases as function of time. As a result, the coherent measuring time, T_{coh}, plays an important role in determining the laser phase noise requirements. Note that for some SAL applications, the coherent measuring time is on the order of 5 msec.

In order to determine the frequency stability of a laser required to meet a specific phase noise requirement over the coherent measuring time, consider the relation between phase and frequency

$$f(t) \equiv \frac{1}{2\pi}\frac{d}{dt}\phi(t)\,. \qquad (5)$$

It follows that the power spectrum of the frequency distribution as a function of the phase distribution is given by

$$S_f(\omega) = \left(\frac{\omega}{2\pi}\right)^2 \cdot S_\phi(\omega)\,. \qquad (6)$$

One can also define the linewidth of a laser in terms of the standard deviation as

$$\Delta f^2 \equiv \sigma_f^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)d\omega\,. \qquad (7)$$

The rms phase error is then given by

$$\phi_o{}^2 \equiv \sigma_\phi^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_\phi(\omega)d\omega\,, \qquad (8)$$

In practice, the absolute phase, or phase offset, is not of concern, and only the phase change during the coherent measuring time is important. We are thus interested in the relative rms phase error during the coherent measurement time. The average phase during a period T is given by

$$\bar{\phi}(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} \phi(\alpha)\cdot d\alpha\,. \qquad (9)$$

The operation of averaging in the interval T is often referred to as smoothing as it removes high-frequency content. It follows that $\bar{\phi}(t)$ can be written as the output of a linear system with $\phi(t)$ as the input and an impulse response, h(t), corresponding to a rectangular pulse. We have

$$\bar{\phi}(t) = \phi(t) \otimes h(t)\,, \qquad (10)$$

where

$$h(t) = \begin{cases} 1/T & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}. \qquad (11)$$

Writing this expression in the frequency domain gives

$$H(j\omega) = \frac{\sin(\omega T/2)}{(\omega T/2)}\,. \qquad (12)$$

The power spectrum of the smoothed signal is then given by the following expression:

$$S_{\bar{\phi}}(\omega) = S_\phi(\omega)\cdot\frac{\sin^2(\omega T/2)}{(\omega T/2)^2}\,. \qquad (13)$$

The phase noise of interest is the rms deviation from the average phase which is given by $y(t) = \phi(t) - \bar{\phi}(t)$. The power spectrum for this phase error is then given by the expression:

$$S_y(\omega) = S_{\phi e}(\omega) - S_{\bar{\phi}}(\omega) = S_{\phi e}(\omega) - S_{\phi e}(\omega)\cdot\frac{\sin^2(\omega T/2)}{(\omega T/2)^2}$$

$$= S_{\phi e}(\omega)\cdot\left(1 - \frac{\sin^2(\omega T/2)}{(\omega T/2)^2}\right)$$

. (14)

Inserting Eq. (4) into Eq. (14) gives

$$S_y(\omega) = S_\phi(\omega)\cdot\left(1 - \frac{\sin^2(\omega T/2)}{(\omega T/2)^2}\right)\cdot\left(4\sin^2(\omega\tau_0/2)\right)$$

. (15)

From Eq. (15) we observe that the power spectrum of the differential phase, y(t), is the power spectrum of the MO laser $\phi(t)$ multiplied by the 2 filter functions in the parentheses in Eq. (15). These filter functions act as a high-pass filters, indicating that static or slow changes in phase have diminishing consequence. The two roll-off frequencies are determined by the coherent processing time, T, and the round trip time τ_0, whichever is longer. As a result, slow phase and frequency drifts do not affect the phase error in y(t).

The filter functions plotted in Figs. (1) and (2) also have the general shape of a high-pass filter.

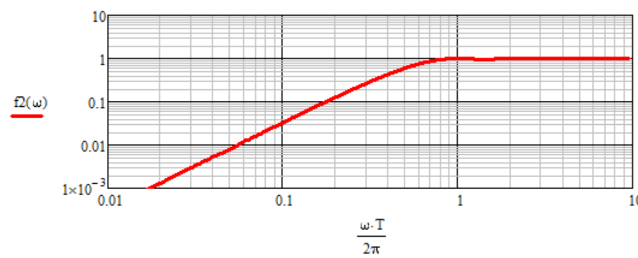


Figure 1. Filter function of the phase power spectrum when only the phase change from the average during the coherent interval T is important.

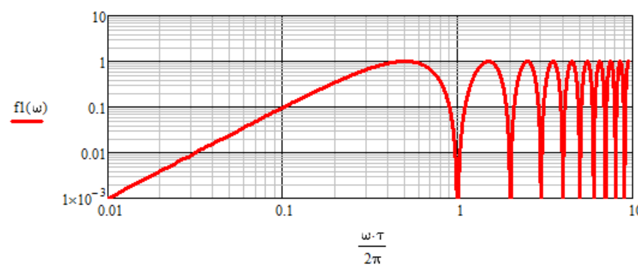


Figure 2. Filter function of the phase power spectrum when single MO laser is used for both transmitter and LO

As noted above the two filter functions of Eq. (15) reduce the impact of phase noise that occurs at low frequencies (slowly changing phase). The first term within parentheses is due to the coherent measurement time, T, and rolls off at a frequency on the order of 1/T. The second term in parentheses is due to the time difference, τ_0, between the LO and return signal and rolls off at a frequency roughly $1/\tau_0$. Because T is typically long, we can make a conservative assumption that the first term in the parentheses is unity for most of the significant phase noise frequencies.

However, for coherent ladar systems to work properly, the high frequency phase noise terms (usually due to acoustics) must be kept under control. In order for the total integrated phase noise to be small, we require that $\omega\tau_0/2 \ll 1$. This simplifies Eq. (15) to

$$S_y(\omega) = S_\phi(\omega)\cdot(\omega\tau_0)^2 . \qquad (16)$$

We then solve for the relevant rms phase noise, σ_y, from equation (16),

$$\sigma_y^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_y(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_\phi(\omega)\cdot(\omega\tau_0)^2 d\omega$$

$$= (2\pi)^2 \tau_0{}^2 \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)d\omega = (2\pi)^2 \tau_0{}^2 \sigma_f^2 = (\tau_0 \cdot 2\pi\Delta f_L)^2 . \qquad (17)$$

Solving for the frequency stability of the laser, we arrive at

$$\Delta f_L = \frac{\sigma_y}{2\pi\tau_0} \qquad (18)$$

As an example, if we need a measurement time of 5 msec with a phase error σ_y of less than $\pi/30$ with an operating range of 15 km, τ_0 is then 100 μsec. Eq. (18) yields

$$\Delta f_L = 167\ \text{Hz}.$$

Note that if we did not use a common MO laser for the transmitter and LO, then we would have to use the longer coherent integration time of 5 msec in Eq.(18) yielding a frequency stability requirement of ~ 3 Hz which is a factor of 50 more severe than for the common case.

Frequency Errors for Single Master Oscillator, MO, for Transmitter and LO Laser (μDoppler)

Frequency error is a more critical parameter when calculating the sensitivity of a micro-Doppler or vibrometry ladar. For this case we need an expression for the frequency error of the measurement as a function of the laser frequency stability, Δf_L. The analysis and derivation are similar to those above, however we now use frequency rather than phase. For the case of a single MO source used for both the transmitter and LO lasers, the frequency error is given by

$$f_e(t) = f(t+\tau_0) - f(t). \qquad (19)$$

where τ_0 represents the round trip time delay between the transmitted and LO beams and f is the frequency time evolution of the laser. As before, the power spectrum of the frequency error is then

$$S_{fe}(\omega) = 4S_f \cdot \sin(\omega\tau_0/2)^2 . \qquad (20)$$

The instantaneous Doppler frequency measured is the quantity used to evaluate the target frequency evolution and determine the vibration of the skin of the target. We find that the frequency error directly contributes to the noise floor of the vibration measurement. The sensitivity of a vibration ladar is the measure of the vibration noise floor or Noise Equivalent Vibration Velocity (NEVV) which has units of μm/s/√Hz. It follows that the NEVV is given by

$$\text{NEVV} = \frac{\lambda}{2}\sqrt{S_{fe}} = \frac{\lambda}{2}\sqrt{S_f \cdot 4\sin(\omega\tau_0/2)^2} . \qquad (21)$$

Let us assume that the laser frequency noise distribution, shown in Fig. (3), is flat up to a maximum frequency of f_{max}, and then drop towards zero.

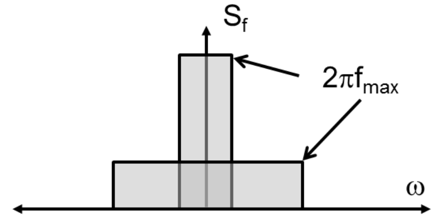


Figure 3. The MO frequency power spectrum is assumed to be a flat distribution.

The variance or rms of the frequency noise is given by

$$(\Delta f_L)^2 \equiv \sigma_f^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)d\omega = 2f_{max}\cdot S_f .$$

Substitution of this result into Eq. (32) yields:

$$\text{NEVV} = \frac{\lambda}{2}\sqrt{S_{fe}} = \frac{\lambda}{2}\cdot\frac{\Delta f_L}{\sqrt{2f_{max}}}\sqrt{4\sin(\omega\tau_0/2)^2} . \qquad (22)$$

In this expression the filtering function of the sin() component is quite useful for eliminating effects from slow frequency drifts. However, as the range of operation increases, it does not reduce the impact of the higher frequency noise. For long range operation, Eq.(22) simplifies to

$$\text{NEVV} = \frac{\lambda}{2}\sqrt{S_{fe}} = \lambda\cdot\frac{\Delta f_L}{\sqrt{2f_{max}}} . \qquad (23)$$

Solving for the laser frequency variance we get

$$\Delta f_L = \frac{1}{\lambda}\text{NEVV}\cdot\sqrt{2f_{max}} . \qquad (24)$$

Consider a long range system that uses a 2μm laser with a frequency noise up to 500 kHz. Assume that for this system we require an NEVV of 100 μm/s/√Hz, then the laser stability needs to be, $\Delta f_L \leq 50$ kHz. Figure (4) shows the frequency stability requirement as function of range delay.

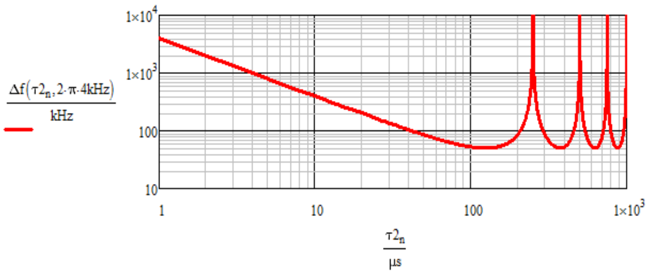


Figure 4. At shorter ranges, Δf_L allowed is larger due to the filtering function, but reaches a minimum of 50 kHz for this example.

The frequency noise spectrum is usually determined by acoustics and RIN noise. The latter typically appears between 500 kHz to 1 MHz

Optical Delay Line Reference to Reduce Laser Stability Requirement

Coherent ladar sensors often need to perform in a stressing scenarios where the coherent processing times are relatively long (multi-milliseconds) and distances to target are also long. Delay times of 100's of microseconds are required. Calculations with Eq. (18) for this scenario show that the laser frequency stability is less than 100 Hz which is difficult to meet in practice.

To relax the frequency stability requirements for the laser we would like to reduce the relative delay value of τ_0 from > 100 µs down to 1-2 µs. The relative time difference of the received and LO signal and thus the source frequency stability can be reduced by introducing an optical delay-line to offset the LO reference by an amount similar to the round-trip transmit time. Figure 5 shows an example block diagram of how a delay-line can be implemented.

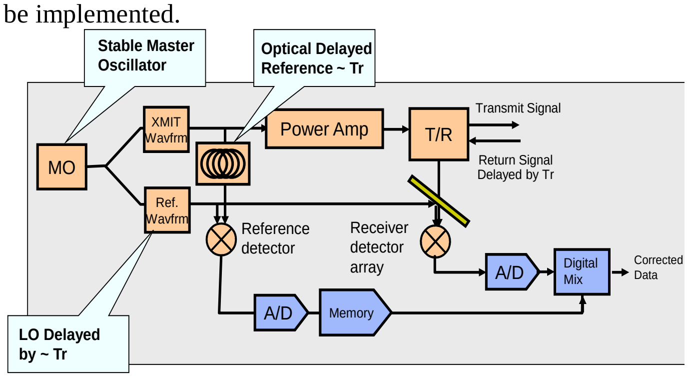


Figure 5. Reference beam and separate LO beam. The delay of the reference can be optimized without limiting the power required for the LO function.

The reference channel is generated by sampling and storing the transmitted chirp waveform using an optical delay line with delay time approximately equal to the round trip time. The sampled and delayed transmitter signal is heterodyne detected with the same LO signal used for target signal detection. This generates a reference signal to be used for phase error correction. The reference beat signal has a relatively low bandwidth and is digitized by a second relatively low speed A/D.

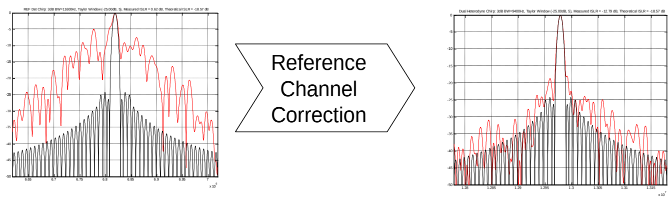
With the above method a relative delay of less than 1 µs (150 m range) can readily be achieved. The frequency stability requirement in Eq. (18) then becomes

$$\Delta f_L = 16.7\,\text{kHz}$$

which is obtainable with fiber oscillators.

Experimental Demonstration

Experimental data was obtained using a setup similar to that shown in Fig. 5 with the round trip to the target simulated by an optical fiber delay. The de-chirped signal at the detector is shown in Fig. (6).



(a)

Figure 6. Compressed signal with and without use of reference correction. (a) Shows the signal uncorrected by the reference and (b) shows the corrected signal. Both traces are super-imposed over the ideal signal predicted by theoretical.

Summary

In this paper we have presented an analysis of the phase noise for a coherent heterodyne ladar. The treatment allows researchers to determine the frequency stability of a transmitter laser required to obtain a desired ladar performance level. We showed that when uncompensated, the frequency stability requirements for a typical scenario can be difficult to achieve, however if we use a phase reference such as a delay version of the transmitted signal, then the frequency stability requirements on the laser and modulator driver become achievable. Furthermore, we showed that the LO and reference channels can be separated and optimized independently. Experimental results shown in Fig. (5) demonstrate the utility of this approach.

[1] **A. Papoulis, "Probability, Random Variables, and Stochastic Processes", McGraw-Hill, 1965**